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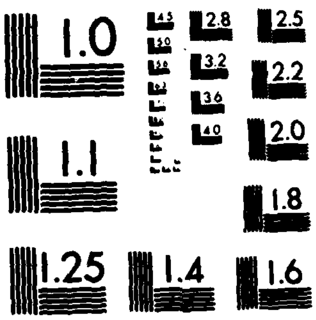

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Research efforts during this period concentrated on the following topics: A. Deformation of Solids - Progress on this topic is written as a separate paper. Currently, investigators are working on the quantum mechanical issues that will determine the transition rates for the crack growth process. B. Markov and Semimarkov Models of Deterioration - With high-reliability devices, the time and expense needed for life tests are prohibitive, and techniques like accelerated life testing and using censored data introduce difficulties of another sort. In an effort to circumvent such difficulties, the concept of a deterioration process is introduced, and the lifetime is defined to be the hitting time of a threshold value by the deterioration process. C. Regenerative Systems and Markov Additive Processes - A system is called regenerative if it enjoys the strong Markov property at stopping times that belong to a certain random time set (called the regeneration set). If the strong Markov property is strengthened to independence of past and future at stopping times belonging to a regeneration set, the system is said to be strictly regenerative. (CONTINUE

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ITEM #19, CONTINUED: In the strict regeneration case, the regeneration set is the image of an increasing Levy process (this is due to MAISONNEUVE), and conversely, the image of an increasing Levy process is a strict regeneration set (this is due to MEYER). A paper on work resulting in a complete characterization of regenerative systems in terms of Markov additive processes is completed. D. Excursions of Markov Processes. Two papers emerged from the work in this area, one gives a simple condition for the mixture of a family of Markovian laws to be Markovian, and then uses the result to derive generalizations to J. Pitman's 2M-X theorem and D. Williams' path decomposition theorem; the other paper re-works the fundamental work of Ito and D. Williams on path decompositions from the point of view of Levy: instead of the maxima of excursions, the lengths of excursions are made the primary objects. This yields more intuitive descriptions of the excursion laws. AND E. Brownian Motion on Riemannian Manifolds. Work in this area has focused on local properties of Brownian motion on Riemannian manifolds. This report summarizes the progress made and lists completed publications resulting from the research.

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PROGRESS REPORT

on

AFOSR GRANT NO. AFOSR 82-0189

MARKOV PROCESSES

APPLIED TO CONTROL, REPLACEMENT, AND SIGNAL ANALYSIS

for the period

1 June 1982 - 31 May 1983

Principle Investigator

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Northwestern University, Evanston, Illinois

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A. DEFORMATION OF SOLIDS

Our progress on this topic is written as a separate report, which is attached. Currently, we are working on the quantum mechanical issues that will determine the transition rates for the crack growth process.

B. MARKOV AND SEMIMARKOV MODELS OF DETERIORATION

With high-reliability devices, the time and expense needed for life tests are prohibitive, and techniques like accelerated life testing and using censored data introduce difficulties of another sort. In an effort to circumvent such difficulties, the concept of a deterioration process is introduced, and the lifetime is defined to be the hitting time of a threshold value by the deterioration process.

The attached paper on this topic discusses the explicit structure of some general models for deterioration processes and solves for the lifetime distribution in general. Using the recent characterization theorems obtained by ÇINLAR and JACOD, the deterioration processes are described in terms of a "deterioration process in intrinsic time" and an "actual time process" as a function of the intrinsic time. This enables us to parameterize fairly general processes in terms of deterministic functions that can be obtained by laboratory tests.

C. REGENERATIVE SYSTEMS AND MARKOV ADDITIVE PROCESSES

A system is called regenerative if it enjoys the strong Markov property at stopping times that belong to a certain random time set (called the

regeneration set). If the strong Markov property is strengthened to independence of past and future at stopping times belonging to a regeneration set, the system is said to be strictly regenerative. In the strict regeneration case, the regeneration set is the image of an increasing Lévy process (this is due to MAISONNEUVE), and conversely, the image of an increasing Lévy process is a strict regeneration set (this is due to MEYER). For the general case of regeneration, the role of the Lévy system is taken over by Markov additive processes. This was shown by JACOD under the assumption that the regeneration set has no isolated points. In joint work with Haya KASPI, we now have a complete characterization of regenerative systems in terms of Markov additive processes. The galley proof of the paper is attached.

D. EXCURSIONS OF MARKOV PROCESSES

This was joint work with Paavo SALMINEN who was supported as a post-doctoral fellow by this grant. Two papers by SALMINEN on this topic are attached.

The one titled "Mixing Markovian laws, with an application to path decompositions" gives a simple condition for the mixture of a family of Markovian laws to be Markovian, and then uses the result to derive generalizations to J. PITMAN's 2M-X theorem and D. WILLIAMS's path decomposition theorem.

The other paper by SALMINEN, "Brownian excursions revisited," the fundamental works of ITO and D. WILLIAMS on path decompositions are re-done from the point of view of LÉVY's: instead of the maxima of excursions, the lengths of excursions are made the primary objects. This yields more intuitive descriptions of the excursion laws.

E. BROWNIAN MOTION ON RIEMANNIAN MANIFOLDS

This work was carried out by Mark Pinsky. It has focused on local properties of Brownian motion on Riemannian manifolds. The specific problem areas under investigation are the following:

a) Mean exit time of Brownian motion from small geodesic balls.

b) Higher moments of the exit time from small geodesic balls.

c) First eigenvalue of the Laplacian on a small geodesic ball.

d) Mean exit time of Brownian motion from tubes about a curve.

The first problem has been solved in collaboration with Alfred Gray, in a paper to appear in the Bulletin des Sciences Mathematiques. The second problem has been solved in a paper to appear in the Proceedings of the International Symposium in honor of Laurent Schwartz. We now describe the setting of these works.

(M,g) is an n-dimensional Riemannian manifold and Δ is the Laplace-Beltrami operator which is canonically associated to the metric g. Using stochastic differential equations we may construct the Brownian motion (X_t, P_x), a conservative strong Markov process with infinitesimal generator Δ. The exit time from a ball is defined as $T_\varepsilon^g = \inf\{t>0: d(X_t,m) = \varepsilon$ where d is the Riemannian distance and m M. This random variable has the scaling property $T_\varepsilon^{cg} = c\, T^g_{\varepsilon/\sqrt{c}}$ in the sense of probability law for any $c > 0$.* This implies that we have the following limit theorem: $T_\varepsilon^g/\varepsilon^2$ has a limiting probability distribution when $\varepsilon \to 0$, independent of the metric g. This is the exit time distribution of Euclidean Brownian motion from the unit ball.

A more detailed study reveals the geometric structure of the manifold from the distribution of the exit time. Let $E_m(T_\varepsilon)$ and $E_m(T_\varepsilon^2)$ be the first and second moments of the exit time. We have the following asymptotic expansions

*Pointed out to us by C. Lmch.

when $\varepsilon \to 0$:

$$E_m(T_\varepsilon) = c_0\varepsilon^2 + c_1\tau\varepsilon^4 + \varepsilon^6[c_2|R|^2 + c_3|\rho|^2 + c_4\tau^2 + c_5\Delta\tau] + O(\varepsilon^8)$$

$$E_m(T_\varepsilon^2) = d_0\varepsilon^4 + d_1\tau\varepsilon^6 + \varepsilon^8[d_2|R|^2 + d_3|\rho|^2 + d_4\tau^2 + d_5\Delta\tau] + O(\varepsilon^{10})$$

where $c_0, \ldots, d_5$ depend only on $n = \dim M$, τ is the scalar curvature, ρ is the Ricci tensor, and R is the Riemann curvature tensor. The constants satisfy $c_2c_3 < 0$, $d_2d_3 < 0$. From these expansions one deduces the following results on retrieving the knowledge of the metric g from the Brownian motion: if the Riemannian manifold (M,g) satisfies $E_m(T_\varepsilon^g) = c_0\varepsilon^2$ for all $m \in M, \varepsilon > 0$ and $2 \le n \le 5$, then g is flat metric. Similar results are available for any metric of constant curvature as well as the other rank one symmetric spaces CP^n, QP^n and CaP^8.

The methods used are a combination of stochastic analysis and Riemannian geometry. We begin with Dynkin's formula and its extension, the so-called "stochastic Taylor formula." These are written in the form

$$E_m f(X_{T_\varepsilon}) - f(m) = E_m \int_0^{T_\varepsilon^g} \Delta f(X_s)ds$$

$$E_m f(X_{T_\varepsilon}) - f(m) = E_m T_\varepsilon^g \Delta f(X_{T_\varepsilon}) - E_m \int_0^{T_\varepsilon^g} sL^2 f(X_s)ds$$

In particular the first moment $u_1 = E_m(T_\varepsilon^g)$ satisfies the equation $u_1 = -1$ while the function $u_2 = \frac{1}{2} E_m(T_\varepsilon^g)^2$ satisfies the equation $\Delta u_2 = -u_1$ with both $u_1 = u_2 = 0$ on the boundary of the ball. To study the Laplacian in a small ball, we develop the following expansion in a system of normal

coordinates:

$$\Delta = \Delta_{-2} + \Delta_0 + \Delta_1 + \cdots$$

where Δ_{-2} is the Laplacian in the tangent space at m and the operators $\{\Delta_j\}_{j\geq 0}$ are second order differential operators with polynomial coefficients. This is used to develop a suitable "perturbation theory" for the Laplacian on a small geodesic ball of a Riemannian manifold. The successive correction terms to the Euclidean mean exit time are expressed as solutions of certain inhomogeneous Poisson equations relative to the Euclidean Laplacian Δ_{-2} where the inhomogeneous terms are expressed in terms of curvature information through the operators $\{\Delta_j\}_{j\geq 0}$. The validity of this perturbation series is justified totally within the probabilistic framework, by repeated use of Dynkin's formula and its extensions.

The third problem also fits into the above context, where we look for the solution $(\lambda_\varepsilon, f_\varepsilon)$ of the problem $\Delta f + \lambda f = 0$ where f is zero on the boundary of the ε-ball and is positive on the interior. It is proved that we have an expansion of the type

$$\lambda_\varepsilon = \alpha_0\varepsilon^{-2} + \alpha_1\tau + \varepsilon^2[\alpha_2|R|^2 + \alpha_3|\rho|^2 + \alpha_4\tau^2 + \alpha_5\Delta\tau] + O(\varepsilon^4)$$

where $\alpha_0, \ldots\ldots, \alpha_5$ depend only on $n = \dim M$. This may be used to recover the metric from knowledge of the fundamental frequency of a geodesic ball in low dimensions.

The fourth problem concerns the exit time of Brownian motion about a curve or submanifold of Euclidean space. We develop an expansion of the Laplacian in a system of "transverse coordinates." This leads to a development of the mean

exit time in an asymptotic power series, where the coefficients are certain integrated curvature invariants. It is shown for example, that we may recognize a circle or a closed geodesic from knowledge of the mean exit time of Brownian motion from an ε-tube about a simple closed curve in a flat manifold.